SCHEDA PER IL RECUPERO

CLASSE 4ª – SOLUZIONE ESPONENZIALI

① $3^x + 9^x = 90$ $\quad 3^x + 3^{2x} - 90 = 0, \quad 3^{2x} + 3^x - 90 = 0 \quad 3^x = t$

$t^2 + t - 90 = 0 \quad t_{1,2} = \dfrac{-1 \pm \sqrt{1+360}}{2} = \begin{cases} + \; t_1 = \dfrac{-1+19}{2} = 9 \\ - \; t_2 = \dfrac{-1-19}{2} = -10 \end{cases}$ $\quad 3^x = 9 \to 3^x = 3^2 \to x = 2$

$3^x = -10$ IMPOSSIBILE

$S = \{2\}$

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② $5^{x-2} + 2 \cdot 5^{x-1} = 55 \quad \dfrac{5^x}{5^2} + \dfrac{2}{5} \cdot 5^x = 55 \quad 5^x\left(\dfrac{1}{25} + \dfrac{2}{5}\right) = 55$

$5^x\left(\dfrac{1+10}{25}\right) = 55 \to \dfrac{11}{25} 5^x = 55 \to 5^x = \cancel{55}^{5} \cdot \dfrac{25}{\cancel{11}} \quad 5^x = 125 \quad 5^x = 5^3 \to \boxed{x = 3}$

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③ $2 \cdot 5^x - 3^{x+1} = 24 \cdot 5^{x-2} - 3^{x-2} \to 2 \cdot 5^x - 24 \cdot 5^{x-2} = 3^{x+1} - 3^{x-2}$ (SEPARO LE BASI =)

$2 \cdot 5^x - 24 \dfrac{5^x}{5^2} = 3^x \cdot 3 - \dfrac{3^x}{3^2} \quad 5^x\left(2 - \dfrac{24}{25}\right) = 3^x\left(3 - \dfrac{1}{9}\right)$

$5^x\left(\dfrac{50-24}{25}\right) = 3^x\left(\dfrac{27-1}{9}\right) \quad 5^x \cdot \dfrac{26}{25} = 3^x \cdot \dfrac{26}{9}$ DIVIDO PER 3^x e per $\dfrac{26}{25}$ entrambi i MEMBRI

$\dfrac{5^x}{3^x} = \dfrac{\cancel{26}}{9} \cdot \dfrac{25}{\cancel{26}} \quad \left(\dfrac{5}{3}\right)^x = \left(\dfrac{5}{3}\right)^2 \to \boxed{x = 2}$

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④ $\dfrac{\sqrt[3]{2^{2-x}}}{\sqrt{2^{3+x}}} = \sqrt[5]{16} \quad \dfrac{2^{\frac{2-x}{3}}}{2^{\frac{3+x}{2}}} = 2^{\frac{4}{5}} \to 2^{\frac{2-x}{3} - \frac{3+x}{2}} = 2^{\frac{4}{5}}$

$\boxed{\text{N.B.} \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}}$

$a^{f(x)} = a^{g(x)} \iff f(x) = g(x)$ poiché l'esponenziale è STRETTAMENTE CRESCENTE (O DECR.) E CONTINUA (quindi INVERTIBILE)

PASSO AGLI ESPONENTI

$\dfrac{2-x}{3} - \dfrac{3+x}{2} = \dfrac{4}{5} \quad \cancel{30} \cdot \dfrac{20-10x-15(3+x)}{\cancel{30}} = \dfrac{24}{\cancel{30}} \cdot \cancel{30}$

$20 - 10x - 45 - 15x = 24 \to -25x = 24 - 20 + 45 \to -25x = 49$

$\boxed{x = -\dfrac{49}{25}}$

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⑤ $\sqrt[3x-1]{2^{2x+1}} = \sqrt[6x-2]{8^{x-2}}$ $\quad 3x-1 \to$ NATURALE, $6x-2 \to \in \mathbb{N}$

C.E. $x \neq \dfrac{1}{3}$, $x \neq \dfrac{2}{6} \neq \dfrac{1}{3}$

$2^{\frac{2x+1}{3x-1}} = 2^{\frac{3(x-2)}{6x-2}} \quad \dfrac{2x+1}{3x-1} = \dfrac{3(x-2)}{\underset{2(3x-1)}{6x-2}}$

$\cancel{2(3x-1)} \dfrac{2 \cdot (2x+1)}{\cancel{2(3x-1)}} = \dfrac{3x-6}{\cancel{2(3x-1)}} \cancel{2(3x-1)} \to 4x + 2 = 3x - 6 \to 4x - 3x = -2 - 6 \quad \boxed{x = -8}$

⑥ $\left(\dfrac{1}{5}\right)^{3-5x^2} = \dfrac{1}{25} \quad \left(\dfrac{1}{5}\right)^{3-5x^2} = \left(\dfrac{1}{5}\right)^2 \quad 3 - 5x^2 = 2 \to -5x^2 = -1 \quad x^2 = \dfrac{1}{5}$

$x = \pm\sqrt{\dfrac{1}{5}} = \pm \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \boxed{\pm \sqrt{5}/5}$

SCHEDA PER IL RECUPERO
CLASSE 4ª – SOLUZIONE ESPONENZIALI LOGARITMI

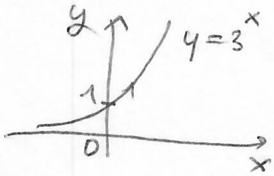
(1) $3^{1+x} + 3^{2+x} - 3^{x-1} - 105 \geq 0$

$3 \cdot 3^x + 3^2 \cdot 3^x - \frac{3^x}{3} \geq 105$ $\qquad 3^x\left(3+9-\frac{1}{3}\right) \geq 105$

$3^x\left(\frac{9+27-1}{3}\right) \geq 105$ $\qquad 3^x \cdot \frac{35}{3} \geq 105$ $\qquad 3^x \geq \cancel{105}^{\,3} \cdot \frac{3}{\cancel{35}_{\,1}}$

$3^x \geq 9 \rightarrow 3^x \geq 3^2$ $\qquad$ Poichè la base è >1 conservo il verso $\geq$

$\boxed{x \geq 2}$



funzione crescente

$a^{x_1} < a^{x_2} \Leftrightarrow x_1 < x_2$

STESSO VERSO

(2) $\left(\left(\frac{2}{3}\right)^{-x-3}\right)^x > \left(\frac{4}{9}\right)^3 \cdot \left(\frac{8}{27}\right)^{x+1}$

$\frac{2}{3}^{(-x-3)x} > \left[\left(\frac{2}{3}\right)^2\right]^3 \cdot \left[\left(\frac{2}{3}\right)^3\right]^{x+1} \rightarrow \left(\frac{2}{3}\right)^{-x^2-3x} > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{3x+3}$

$\rightarrow \quad -x^2 - 3x < 6 + 3x + 3$

$-x^2 - 3x - 3x - 9 < 0$

$-x^2 - 6x - 9 < 0$

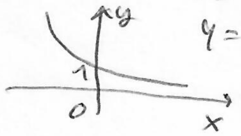
$x^2 + 6x + 9 > 0$

$x_{1,2} = \frac{-6 \pm \sqrt{36-36}}{2} = -\frac{6}{2} = -3$

(N.B. $x^2+6x+9=(x+3)^2$)

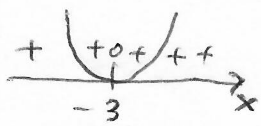
CAMBIO IL VERSO POICHE' LA BASE E' $0<\frac{2}{3}<1$

FUNZIONE DECRESCENTE



$a^{x_1} < a^{x_2} \Leftrightarrow x_1 > x_2$

CAMBIO VERSO



$\forall x \in \mathbb{R}, x \neq -3$

(3) $2\log_3 x - 2 \leq 0$ $\qquad \boxed{\text{C.E. } x > 0}$

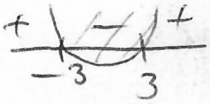
$2\log_3 x \leq 2$ $\qquad \log_3 x^2 \leq 2$ $\qquad$ con la def. $x^2 \leq 3^2$

MANTENGO IL VERSO PERCHE' IL $\log_3$ E' CRESCENTE (BASE >1)

OPPURE $\quad \log_3 x^2 \leq \log_3 3^2 \qquad x^2 \leq 9 \qquad x^2 - 9 \leq 0$

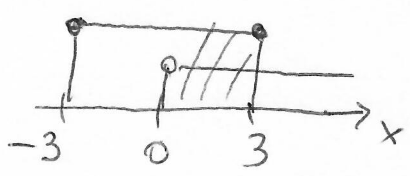
PASSO ALL'EQUAZIONE ASSOCIATA

$x^2 = 9 \qquad x = \pm 3$



$\boxed{-3 \leq x \leq 3}$

C.E.



S: $\boxed{0 < x \leq 3}$

$\left[\begin{cases} x > 0 & \text{C.E.} \\ \log_3 x^2 \leq 2 \end{cases}\right.$ DEVO RISOLVERE UN SISTEMA CON IL C.E.

④ $\log_2(x-2) - \log_2(x-3) > 1$

$1 = \log_2 2$

C.E
C.E.
PROPRIETA' DELLA DIFFERENZA DI LOG.

$$\begin{cases} x-2>0 \\ x-3>0 \\ \log_2 \dfrac{(x-2)}{x-3} > \log_2 2 \end{cases} \qquad \begin{cases} x>2 \\ x>3 \\ \dfrac{x-2}{x-3} > 2 \end{cases}$$

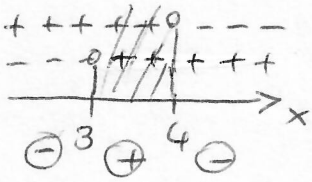
MANTENGO IL VERSO BASE >1

$$\begin{cases} x>2 \\ x>3 \\ \dfrac{x-2}{x-3} - 2 > 0 \end{cases} \rightarrow \frac{x-2-2(x-3)}{x-3} > 0 \qquad \frac{x-2-2x+6}{x-3} > 0$$

$$\begin{cases} x>2 \\ x>3 \\ \dfrac{-x+4}{x-3} > 0 \end{cases}$$

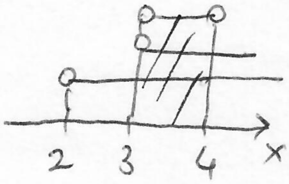
$N>0 \quad -x+4>0 \quad x<4$

$D>0 \quad x-3>0 \quad x>3$



$3<x<4$

$$\begin{cases} x>2 \\ x>3 \\ 3<x<4 \end{cases}$$



$\boxed{S = 3<x<4}$

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⑤ $\log(x-2) + \log(5-x) > \log 2$

$\log x = \log_{10} x = \log$

NOI ABBIAMO USATO QUESTA NOTAZIONE

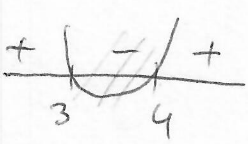
$\log(x-2)(5-x) > \log 2$

$$\begin{cases} x-2>0 \\ 5-x>0 \\ (x-2)(5-x)>2 \end{cases}$$ CONSERVO IL VERSO la BASE (10) > 1

$$\begin{cases} x>2 \\ x<5 \\ 5x-x^2-10+2x-2>0 \end{cases} \qquad \begin{cases} x>2 \\ x<5 \\ -x^2+7x-12>0 \end{cases} \rightarrow x^2-7x+12<0$$

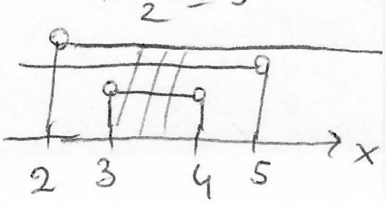
$x^2-7x+12=0$

$$x_{1,2} = \frac{7 \pm \sqrt{49-48}}{2} = \begin{cases} + \quad \dfrac{7+1}{2} = 4 \\ - \quad \dfrac{7-1}{2} = 3 \end{cases}$$



$3<x<4$

$$\begin{cases} x>2 \\ x<5 \\ 3<x<4 \end{cases} \rightarrow$$



$S: \quad 3<x<4$

⑥ $\log_{\frac{1}{2}} x + \log_{\frac{1}{2}} 16 < 1$ C.E. $x > 0$

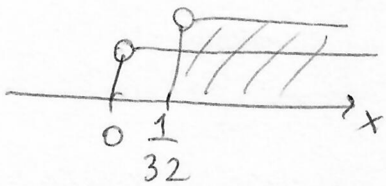
$$\log_{\frac{1}{2}} 16x < \log_{\frac{1}{2}} \frac{1}{2}$$

$1 = \log_{\frac{1}{2}} \frac{1}{2}$

BASE

$0 < \frac{1}{2} < 1 \Rightarrow$ PASSO AGLI ARGOMENTI MA CAMBIO VERSO.

C.E. $\begin{cases} x > 0 \\ 16x > \frac{1}{2} \end{cases}$ $\begin{cases} x > 0 \\ x > \frac{1}{2} \cdot \frac{1}{16} \end{cases}$ $\begin{cases} x > 0 \\ x > \frac{1}{32} \end{cases}$



$$\boxed{S: x > \frac{1}{32}}$$